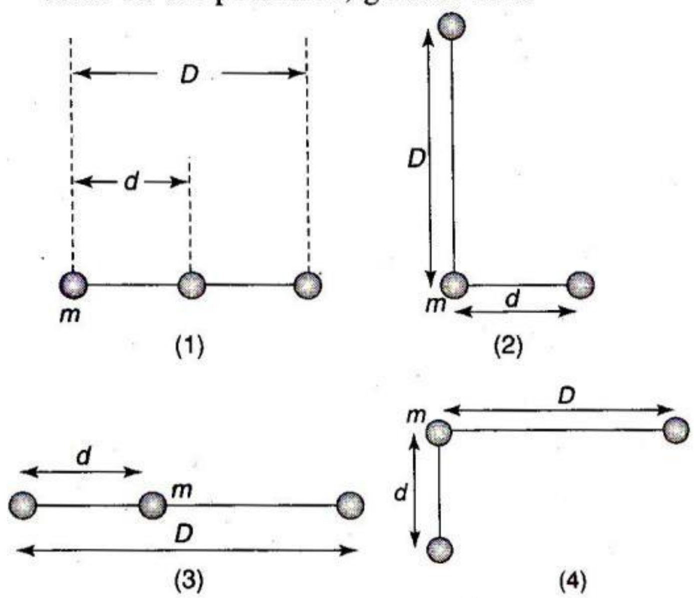
01. The figure shows four arrangements of three particles of equal masses. Rank the arrangement according to the magnitude of the net gravitational force on the particle m, greatest first.



(a) 1, tie of 2 and 4, then 3
(b) 1, 4, 3, 2
(c) 2, 3, 4, 1
(d) 4, 3, 1, 2

02. Three identical point masses, each of mass 1 kg lie in the x-y plane at points (0, 0), (0, 0.2m) and (0.2m, 0). The net gravitational force on the mass at the origin is

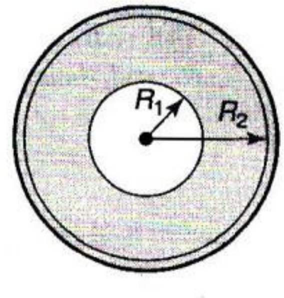
(a) $1.67 \times 10^{-9}(\hat{j} + \hat{j})N$

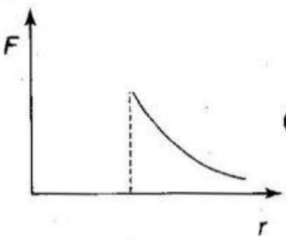
(b) $3.34 \times 10^{-10}(\hat{i} + \hat{j})N$

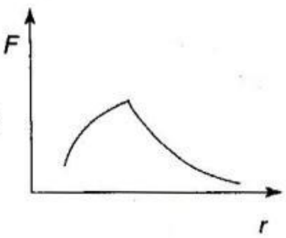
(c) $1.67 \times 10^{-9}(\hat{i} - \hat{j})N$

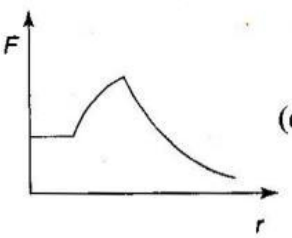
(d) $3.34 \times 10^{-10}(\hat{i} + \hat{j})N$

03. A sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in figure. The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as $(0 \le r \le \infty)$



(a)


(b)


(c)


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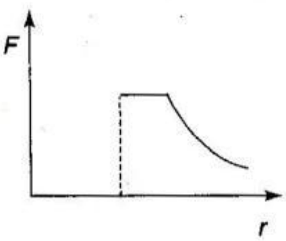


04. Let the minimum external work done in shifting a particle from centre of earth to earth's surface be W_1 and that from surface of earth to infinity be W_2. Then $\frac{W_1}{W_2}$ is equal to

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 3

05. A rocket of mass M is launched vertically from the surface of the earth with an initial speed V. Assuming the radius of the earth to be R and negligible air resistance, the maximum height attained by the rocket above the surface of the earth is

(a) $R/\left(\frac{gR}{2V^2} - 1\right)$ (b) $R\left(\frac{gR}{2V^2} - 1\right)$

(c) $R/\left(\frac{2gR}{V^2} - 1\right)$ (d) $R\left(\frac{2gR}{V^2} - 1\right)$

06. A satellite moves eastwards very near the surface of the Earth in equatorial plane with speed (v_0). Another satellite moves at the same height with the same speed in the equatorial plane but westwards. If R = radius of the Earth and ω be its angular speed of the Earth about its own axis. Then find the approximate difference in the two time period as observed on the Earth.

(a) $\dfrac{4\pi\omega R^2}{v_0^2 + R^2\omega^2}$ (b) $\dfrac{2\pi\omega R^2}{v_0^2 - R^2\omega^2}$

(c) $\dfrac{4\pi\omega R^2}{v_0^2 - R^2\omega^2}$ (d) $\dfrac{2\pi\omega R^2}{v_0^2 + R^2\omega^2}$

07. Two stars each of mass M and radius R are approaching each other for a head-on collision. They start approaching each other when their separation is $r >> R$. If their speeds at this separation are negligible, the speed v with which they collide would be

(a) $v = \sqrt{GM\left(\dfrac{1}{R} - \dfrac{1}{r}\right)}$

(b) $v = \sqrt{GM\left(\dfrac{1}{2R} - \dfrac{1}{r}\right)}$

(c) $v = \sqrt{GM\left(\dfrac{1}{R} + \dfrac{1}{r}\right)}$

(d) $v = \sqrt{GM\left(\dfrac{1}{2R} + \dfrac{1}{r}\right)}$

08. The density of core of a planet is ρ_1 and that of outer shell is ρ_2. The radius of core is R and that of planet is $2R$. Gravitational field at outer surface of planet is same as at the surface of core. What is the ratio ρ_1/ρ_2.

(a) 3/4 (b) 5/3

(c) 7/3 (d) 3/5

09. An asteroid of mass m is approaching earth, initially at a distance 10 R_E with speed v_i. It hits earth with a speed v_f (R_E and M_E are radius and mass of earth), then

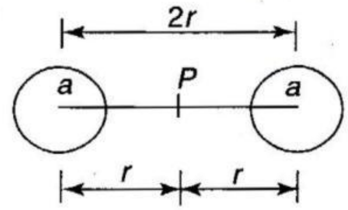
(a) $v_f^2 = v_i^2 + \dfrac{2Gm}{R_E}\left(1 + \dfrac{1}{10}\right)$

(b) $v_f^2 = v_i^2 + \dfrac{2Gm_E}{R_E}\left(1 + \dfrac{1}{10}\right)$

(c) $v_f^2 = v_i^2 + \dfrac{2Gm_E}{R_E}\left(1 - \dfrac{1}{10}\right)$

(d) $v_f^2 = v_i^2 + \dfrac{2Gm}{R_E}\left(1 - \dfrac{1}{10}\right)$

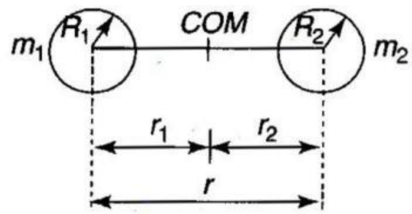
10. A particle is projected from the surface of one star towards other star of same radius a and mass with such a minimum velocity $K\sqrt{\dfrac{GM}{a}}$, so that it is attracted towards other star. Find the value of K if two stars are $2r$ distance apart:



(a) $\dfrac{2(r-a)}{[r-(2r-a)]^{1/2}}$ (b) $\dfrac{2(r-a)}{[r(r-a)]^{1/2}}$

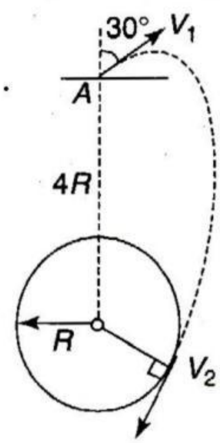
(c) $\dfrac{r-a}{[r-(r-a)]^{1/2}}$ (d) $\dfrac{r+a}{[r-(r-a)]^{1/2}}$

11. Binary stars of comparable masses rotate under the influence of each other's gravity at a distance $\left[\frac{2G}{\omega^2}\right]^{1/3}$ where ω is the angular velocity of each of the systems. If difference between the masses of two stars is 6 units. Find the ratio of the masses of smaller to bigger star.



(a) 4 : 10 (b) 1 : 7
(c) 2 : 8 (d) 3 : 9

12. A particle is projected from point A, that is at a distance $4R$ from the centre of the earth, with speed V_1 in a direction making 30° with the line joining the centre of the earth and point A, as shown. Consider gravitational interaction only between these two. $\left(\text{Use } \frac{GM}{R} = 6.4 \times 10^7\ \text{m}^2/\text{s}^2\right)$.
The speed V_1 if particle passes grazing the surface of the earth is



(a) $2\sqrt{2} \times 10^3$ m/s (b) $4\sqrt{2} \times 10^3$ m/s
(c) 4×10^3 m/s (d) $4\sqrt{3} \times 10^3$ m/s

13. Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is

(a) $\left[2G\frac{(m_1 - m_2)}{r}\right]^{1/2}$ (b) $\left[\frac{r}{2G(m_1 m_2)}\right]^{1/2}$

(c) $\left[\frac{2G}{r}(m_1 + m_2)\right]^{1/2}$ (d) $\left[\frac{2G}{r}m_1 m_2\right]^{1/2}$

14. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The gravitational potential at a point situated at $\frac{a}{2}$ distance from the centre, will be

(a) $-\frac{3GM}{a}$ (b) $-\frac{2GM}{a}$

(c) $-\frac{GM}{a}$ (d) $-\frac{4GM}{a}$

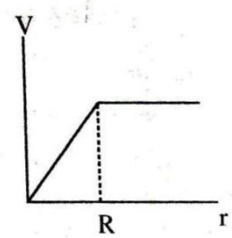
15. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then

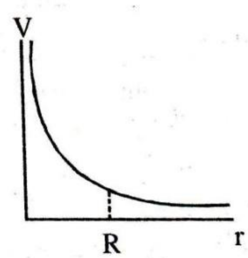
(a) $T_A > T_B$ (if $r_A > r_B$) (b) $T_A > T_B$ (if $m_A > m_B$)

(c) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$ (d) $T_A = T_B$

16. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases}\rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R\end{cases}$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence o the gravitational field of particles. Its speed V as a function of distance r $(0 < r < \infty)$ from the centre of the system is represented by

(a)


(b)


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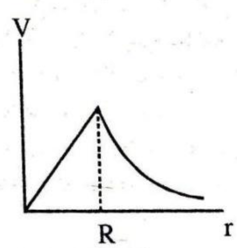
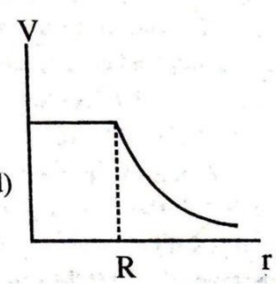


(d)


17. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are)

(a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(d) The energy of the mass m remains constant

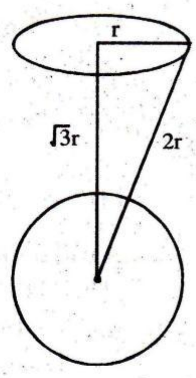
18. Taking the gravitational potential at a point infinte distance away as zero, the gravitational potential at a point A is –5 unit. If the gravitational potential at point infinite distance away is taken as + 10 units, the potential at point A is

(a) – 5 unit (b) + 5 unit

(c) + 10 unit (d) + 15 unit

19. A uniform ring of mass m and radius r is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is directly above the centre of the sphere at a distance $r\sqrt{3}$ as shown in the figure.

The gravitational force exerted by the sphere on the ring will be



(a) $\frac{GMm}{8r^2}$

(b) $\frac{GMm}{4r^2}$

(c) $\sqrt{3}\frac{GMm}{8r^2}$

(d) $\frac{GMm}{8r^3\sqrt{3}}$

20. A planet is revolving around the sun in an elliptical orbit. Its closests distance from the sun is r_{min}. The farthest distance from the sun is r_{max}. If the orbital angular velocity of the planet when it is nearest to the sun is ω, then the orbital angular velocity at the point when it is at the farthest distance from the sun is

(a) $\sqrt{(r_{min}/r_{max})}\,\omega$ (b) $\sqrt{(r_{max}/r_{min})}\,\omega$

(c) $(r_{max}{}^2/r_{min}{}^2)\omega$ (d) $(r_{min}{}^2/r_{max}{}^2)\omega$

21. A projectile is fired vertically from the Earth with a velocity kv_e where v_e is the escape velocity and k is a constant less than unity. The maximum height to which projectile rises, as measured from the centre of Earth, is

(a) $\frac{R}{k}$ (b) $\frac{R}{k-1}$

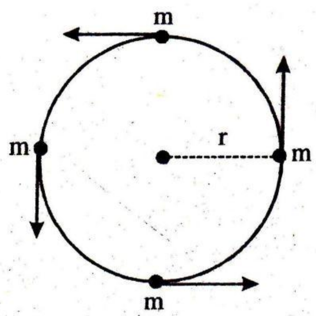
(c) $\frac{R}{1-k^2}$ (d) $\frac{R}{1+k^2}$

22. A man of mass m starts falling towards a planet of mass M and radius R. As he reaches near to the surface, he realizes that he will pass through a small hole in the planet. As he enters the hole, he sees that the planet is really made of two pieces a spherical shell of negligible thickness of mass 2M/3 and a point mass M/3 at the centre. Change in the force of gravity experienced by the man is

(a) $\frac{2}{3}\frac{GMm}{R^2}$ (b) 0

(c) $\frac{1}{3}\frac{GMm}{R^2}$ (d) $\frac{4}{3}\frac{GMm}{R^2}$

23. Four similar particles of mass m are orbiting in a circle of radius r in the same angular direction because of their mutual gravitational attractive force. Velocity of a particle is given by



(a) $\left[\frac{GM}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{1/2}$ (b) $\sqrt[3]{\frac{GM}{r}}$

(c) $\sqrt{\frac{GM}{r}\left(1+2\sqrt{2}\right)}$ (d) $\left[\frac{1}{2}\frac{GM}{r}\left(\frac{1+\sqrt{2}}{2}\right)\right]^{1/2}$

24. A satellite is revolving round the earth in an orbit of radius r with time period T. If the satellite is revolving round the earth in an orbit of radius $r + \Delta r$ ($\Delta r << r$) with time period $T + \Delta T$ then,

(a) $\frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta r}{r}$ (b) $\frac{\Delta T}{T} = \frac{2}{3}\frac{\Delta r}{r}$

(c) $\frac{\Delta T}{T} = \frac{\Delta r}{r}$ (d) $\frac{\Delta T}{T} = -\frac{\Delta r}{r}$

25. The escape velocity from a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(a) v_e (b) $v_e/\sqrt{2}$

(c) $v_e/2$ (d) zero